Abrupt Decorrelation in Ornstein–Uhlenbeck Processes

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Abstract

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The study of the cut-off phenomenon-also known as abrupt convergence (to equilibrium) or abrupt thermalization-has a history spanning several decades, from the seminal work of Aldous and Diaconis (1986) (1) on random card shuffling to its appearance in many modern contexts. In very simplified terms, the cut-off phenomenon refers to the following behavior: starting from a fixed initial distribution (typically a Dirac delta), one evolves under a given dynamics. When measuring the distance between the distribution at time t and the stationary distribution (in some preferred classical metric), one observes that this distance remains essentially maximal for a long time and then drops abruptly to zero on a specific time scale, staying close to zero thereafter.

In our work, we introduce and study a phenomenon that, to the best of our knowledge, has not been previously analyzed on its own within the rigorous mathematical literature, and which we call abrupt decorrelation. We start from a random initial distribution m0. As the dynamics evolves, we observe that the correlation between the distribution at time t, denoted by mt and the initial distribution m0, undergoes an abrupt decay. More explicitly, we consider the distance (in some classical metric) between the joint law of (m0, mt) and the product measure m0 $\times$ mt in order to detect this phenomenon.

In an analogous way as the theoretical framework for the cut-off phenomenon developed by Barrera and Ycart (2014) (2), we propose three levels of abrupt decorrelation (at a sequence of times, with a window, and with a profile). In this talk, we focus on a particularly tractable case where many explicit computations are possible: Ornstein–Uhlenbeck processes.

References

(1) Aldous, D., Diaconis, P. Shuffling cards and stopping times. Amer.

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(2) Barrera, J., Ycart, B. Bounds for left and right window cutoffs. ALEA
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